P5/6 Mathematics Parents' Workshop

7 April 2018

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Mr Nach

What is Heuristics?

Heuristics are general rules of thumb of what students can do to tackle a problem when solutions to the problem is not obvious.

These include:

- Using a representation (eg drawing a diagram, tabulating)
- Making a guess (eg guess and check, making a supposition)
- Walking through the process (eg acting it out, working backwards)
- Changing the problem (eg simplifying the problem)

T & L of Math @ TVPS

- Factual Fluency
- Heuristics is incorporated into the various topics and taught using the
  "I do, We do and You do" approach
- Leveraging on ICT

Factual Fluency

What is factual fluency?

Factual fluency refers to the ability to recall the basic facts in all four operations **accurately, quickly** and **effortlessly**.

What's the big deal with factual fluency?

Through automaticity, students **free up their working memory** and can devote it to problem solving and learning new concepts and skills (Geary, 1994).

P5/P6 Heuristics Approach
Looking for Patterns

WE DO

Look at the pattern below.
M A T H M A T H …

a) What is the 105^{th} letter?

WE DO

Look at the pattern below.
M A T H M A T H …

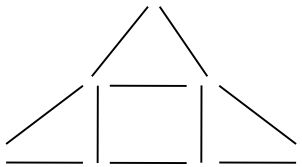
105 ÷ 4 = 26 r 1

The letter is M

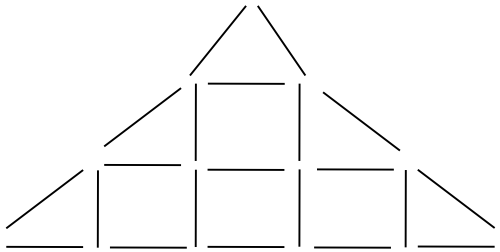
WE DO

The pattern below is made up of toothpicks.

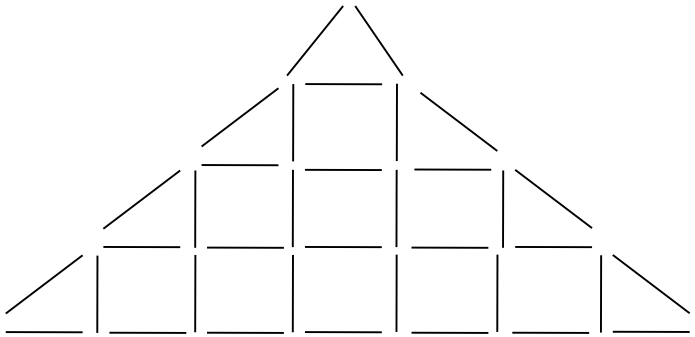
Study the pattern carefully and answer the following questions.



Pattern 1



Pattern 2



Pattern 3

Complete the following table below.

Pattern number	Number of triangles	Number of squares
1	3	1
2	5	4
3	7	9

a) How many triangles are there in Pattern 18?

b) Which pattern has 441 squares?

Solution:

Pattern number	Number of triangles	Number of squares
1	3	1
2	3 + 2 = 5	2 X 2 = 4
3	5 + 2 = 7	3 X 3 = 9
4	7 + 2 = 9	4 X 4 = 16

a) Number of triangles →

(Pattern number × 2) + 1

= (18 x 2) + 1

= 36 + 1

= **37 triangles**

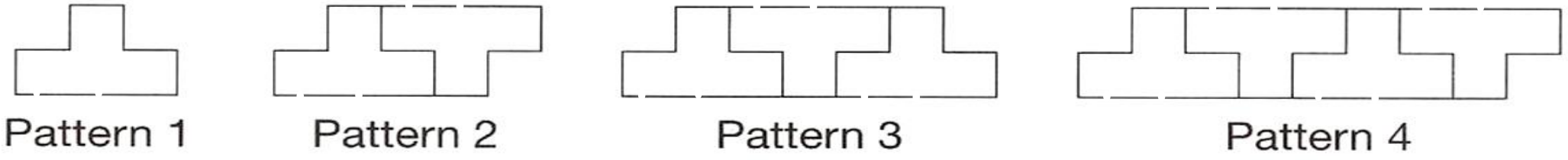
b) Number of squares →

Pattern number x Pattern number

21 x 21 = 421

Pattern 21 has 421 squares.

10-cm sticks are used to make patterns of T-block shapes



WE DO

Pattern No.	No. of T-blocks	Number of 10-cm sticks
1	1	10
2	2	17
3	3	24
4	4	31

(a) How many sticks are used to make Pattern 7?

(b) How many sticks are used to make the pattern with 100 T-blocks?

(c) What is the perimeter of the pattern with 180 T-blocks?

Pattern No.	Number of T-blocks	Number of 10-cm sticks
1	1	10
2	2	17
3	3	24
4	4	31
5	5	38
6	6	45

7

7

7

(a) How many sticks are used to make Pattern 7?

45 + 7 = **52**

Alternative method:

Working ----------(Pattern No. x 7) + 3

= (7 x 7) + 3

= **52**

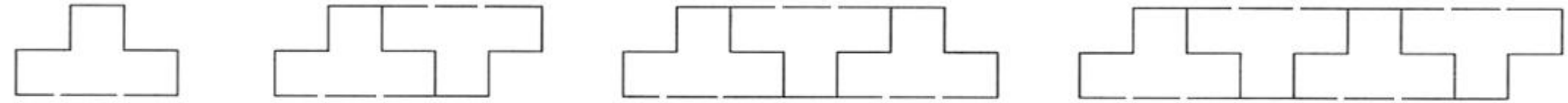
Pattern No.	Number of T-blocks	Number of 10-cm sticks
1	1	10
2	2	17
3	3	24
4	4	31
5	5	38
6	6	45

(b) How many sticks are used to make the pattern with 100 T-blocks?

Formula ----------(Pattern No. x 7) + 3

= (100 x 7) + 3

= **703**



Pattern 1 Pattern 2 Pattern 3 Pattern 4

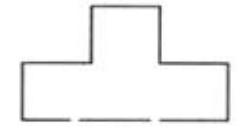
Pattern No.	Number of T-blocks	Number of 10-cm sticks	Perimeter (cm)
1	1	10	10 x 10
2	2	17	14 x 10
3	3	24	18 x 10
4	4	31	22 x 10
5	5	38	26 x 10
6	6	45	30 x 10

4

4

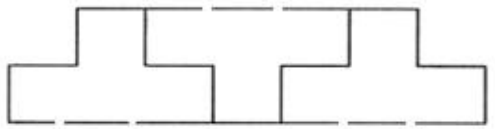
4

(c) What is the perimeter of the pattern with 180 T-blocks?

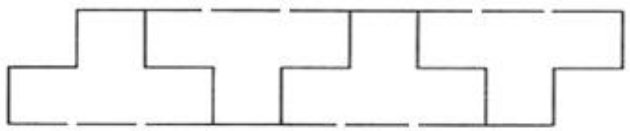


Pattern 1

Pattern 2



Pattern 3



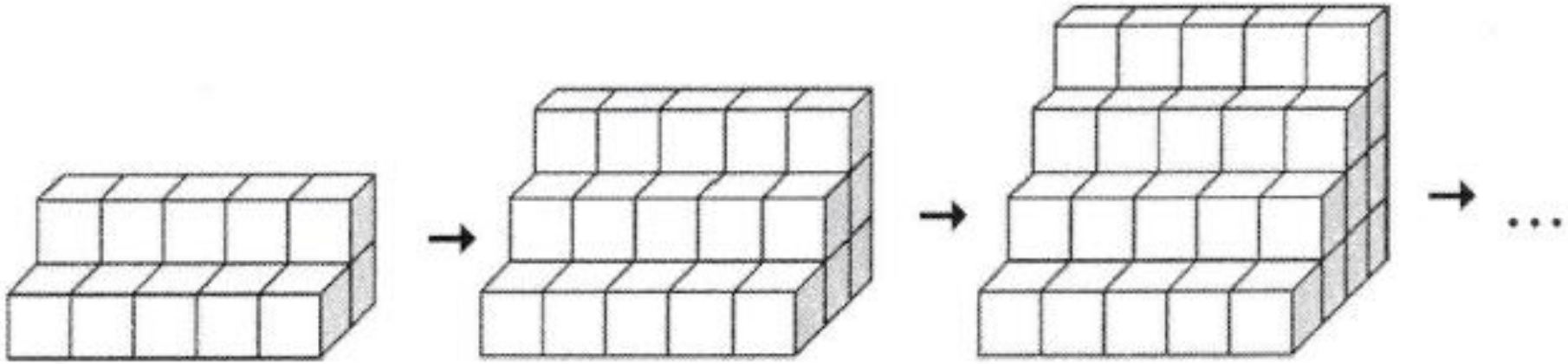
Pattern 4

Pattern No.	Number of T-blocks	Number of 10-cm sticks	Perimeter (cm)	Formula
1	1	10	10 x 10	[(1x 4) + 6] x 10 = 100
2	2	17	14 x 10	[(2 x 4) + 6] x 10 = 140
3	3	24	18 x 10	[(3 x 4) + 6] x 10 = 180
4	4	31	22 x 10	[(4 x 4) + 6] x 10 = 220
5	5	38	26 x 10	[(5 x 4) + 6] x 10 = 260
6	6	45	30 x 10	[(6 x 4) + 6] x 10 = 300

[(180 x 4) + 6] x 10 = 7260

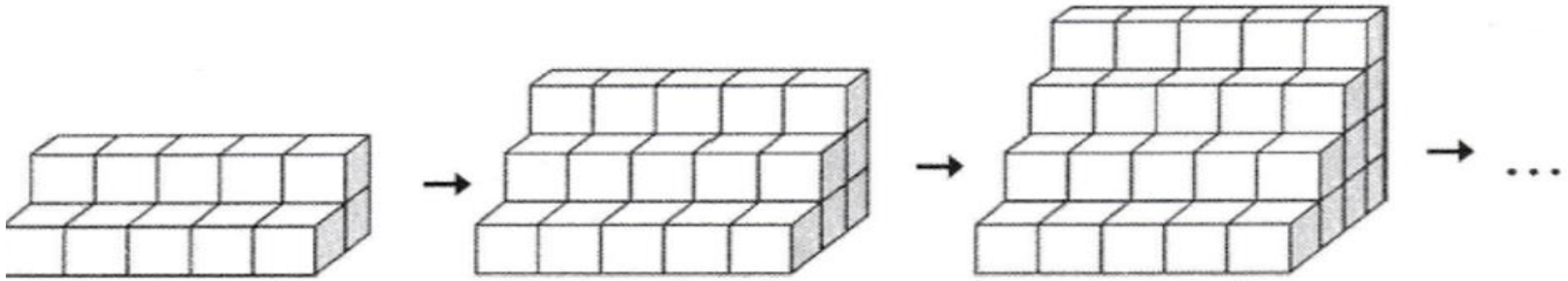
YOU DO

Meng wanted to build a set of steps with 1-cm cubes ( ). The figures below show how he built the steps, from a height of 2 cm to 3 cm to 4 cm.



If Meng continued building the steps in this way, what would be the height of the set of steps that had 140 cubes?

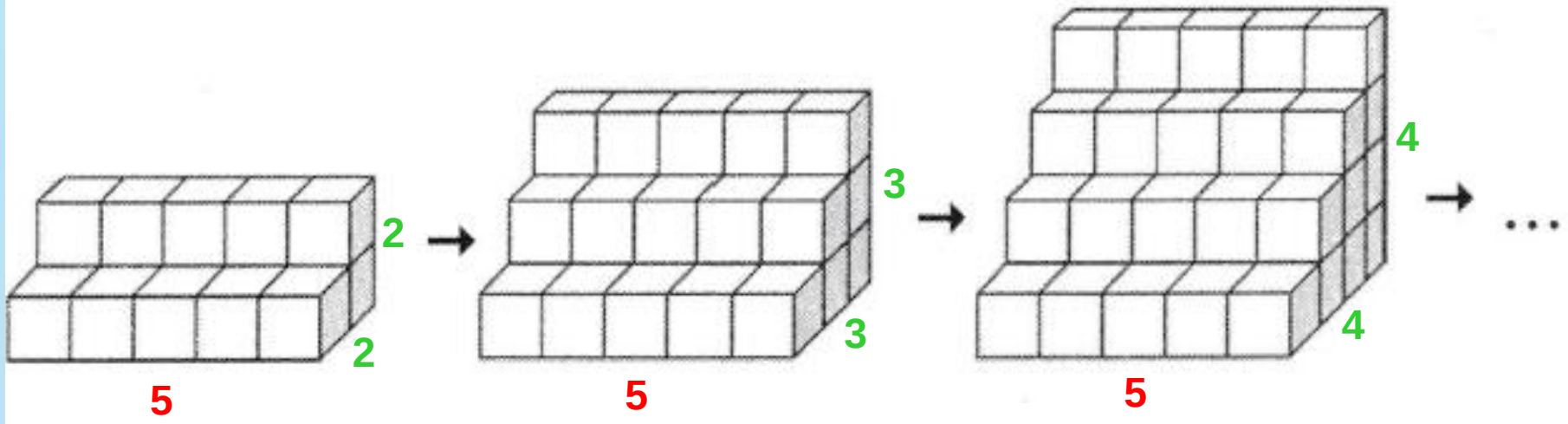
PSLE 2015



If Meng continued building the steps in this way, what would be the height of the set of steps that had 140 cubes?

Figure Number	Height	Number of cubes
1	2	15
2	3	30
3	4	50
4	5	75
5	6	105
6	7	140

(Solution 2)



Observation

Common Characteristic: 5 cubes *(as seen from the front)*

Variations: The lower layer has one cube more than the layer above

(as seen in the cross-section) *1 + 2 + 3 + 4 …*

140 ÷ 5 = 28

1 + 2 + 3 + 4 + 5 + 6 + **7** = 28 (add till the sum is 28)

Height = **7 cm**

No. of cubes in the cross-section at the bottom-most layer is indicative of the height since they are always the same as observed.

YOU DO

The first 15 numbers of a number pattern are given below.

4, 0, 1, 2, 4, 0, 1, 2, 4, 0, 1, 2, 4, 0, 1…

1st 15th

a) What is the 626th number?

b) What is the sum of the first 627 numbers?

PSLE 2017

4, 0, 1, 2, 4, 0, 1, 2, 4, 0, 1, 2, 4, 0, 1…

1st 15th

Each set has 4 numbers.

626 ÷ 4 = 156 r 2

The 626th number is second number after the 156th set.

a) Hence the number is **0**.

b) Sum of each set is 4 + 0 + 1+ 2 = 7

627 ÷ 4 = 156 r 3

(156 x 7) + 4 + 0 + 1 = **1097**

PSLE 2017

P5/P6 Heuristics Approach
Repeated Identity

WE DO

Paul's height is $\frac{2}{5}$ of his father's height and $\frac{3}{4}$ of Sean's height.

a) Find the ratio of Paul's height to the total height of his father and Sean.

b) Sean is 84cm shorter than Paul's father. What is Paul's height in metres?

P5

Paul's height is $\frac{2}{5}$ of his father's height and $\frac{3}{4}$ of Sean's height.

Paul's height is $\frac{2}{5}$ of his father's height

P : F
2 : 5

Paul's height is $\frac{3}{4}$ of Sean's height.

P : S
3 : 4

P : F
2 : 5

x3 x3

P : F
6 : 15

P : S
3 : 4

x2 x2

P : S
6 : 8

P : F : S
6 : 15 : 8

a) Find the ratio of Paul's height to the total height of his father and Sean.

P : F + S
6 : 23

P :	F : S
6 :	15 : 8

b) Sean is 84cm shorter than Paul's father. What is Paul's height in metres?

15 – 8 = 7

7p ----------84

1p ----------12

6p ----------72

72 cm = 0.72m

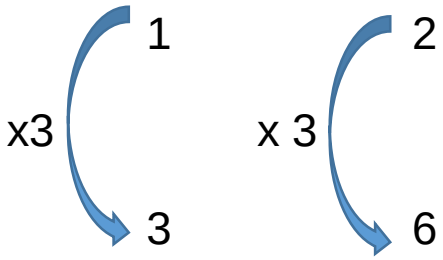
Paul is **0.72 m**

WE DO

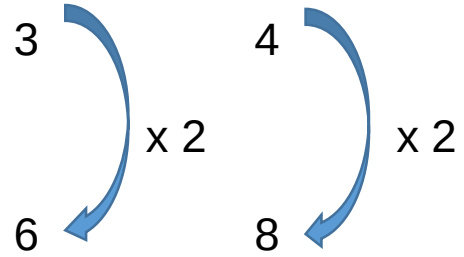
Rachel had $\frac{1}{2}$ as much money as Aaron. Aaron had $\frac{3}{4}$ as much money as Lynn.

If Rachel had $45 less than Lynn, find the sum of money the three of them had.

Rachel : Aaron



Aaron : Lynn



8 u - 3 u = 5 u

5 u = $45

1 u = $45 ÷ 5

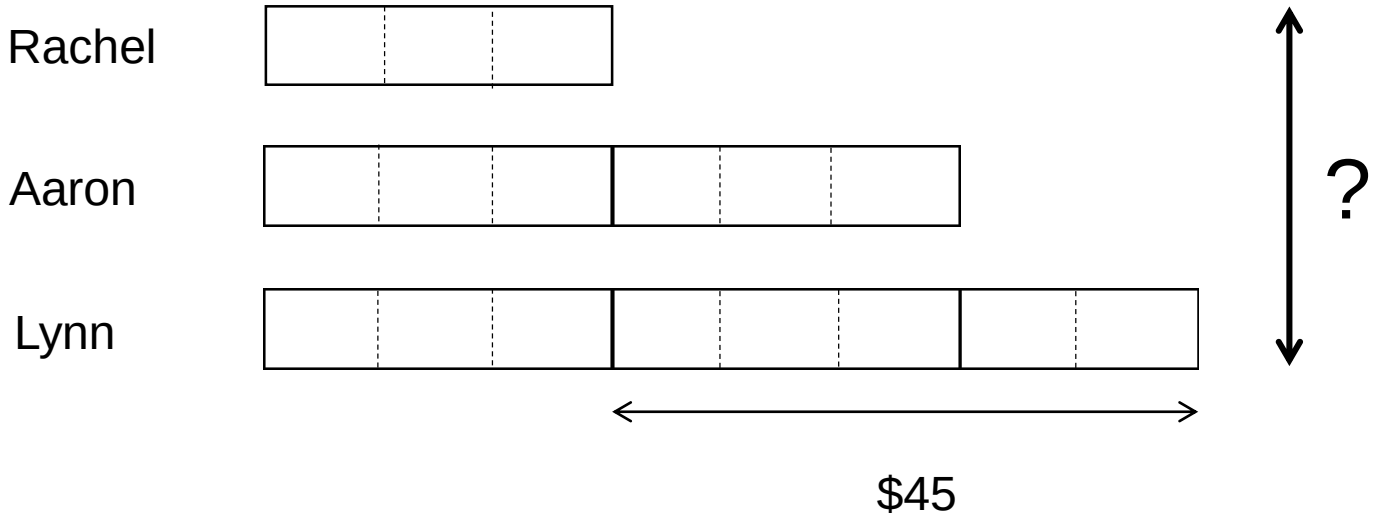
= $9

3 u + 6 u + 8 u = 17 u

17 u = 17 x $ 9

= **$153**

Model Method



5 u = $45

1 u = $45 ÷ 5

= $9

17 u = $9 x 17

= **$153**

YOU DO

A teacher has a number of happy face stickers in three colours: yellow, orange and blue.

$\frac{3}{10}$ of the stickers are yellow.

The number of yellow stickers is twice the number of orange stickers.

What fraction of the stickers are blue?

PSLE 2014

Y : O + B

3 : 7

X 2 X 2

Y : O

2 : 1

X 3 X 3

Y : O + B

6 : 14

Y : O

6 : 3

Y : O : B

6 : 3 : 11

It is $\frac{11}{20}$

YOU DO

A bag contains straws of three colours.

$\frac{1}{4}$ of the straws are blue.

The ratio of the number of red straws to that of green straws is 2 : 3.

What is the ratio of the number of blue straws to that of green straws ?

PSLE 2015

Solution :

Blue	:	Red + Green	Red	:	Green	:	Total
1	:	3	2	:	3	:	5
(x 5)		(x 5)	(x 3)		(x 3)		(x3)
5	:	15	6	:	9		

Blue	:	Green
5	**:**	**9**

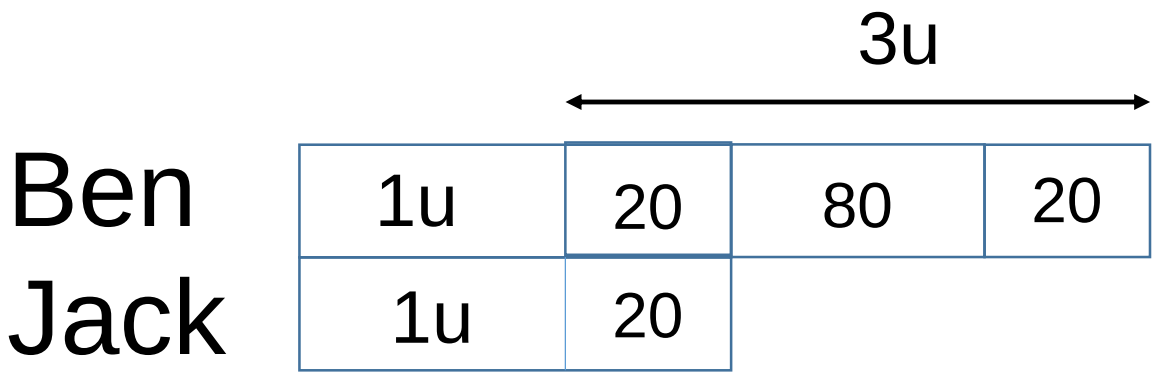
P5/P6 Heuristics Approach

Total Unchanged

WE DO

Ben had 80 more marbles than Jack. After a game, Jack lost 20 marbles to Ben and as a result, Ben had 4 times as many marbles as Jack.

How many marbles did Jack have before the game?



3 u = 120

1 u = 40

40 + 20 = 60

Jack had **60 marbles** before the game.

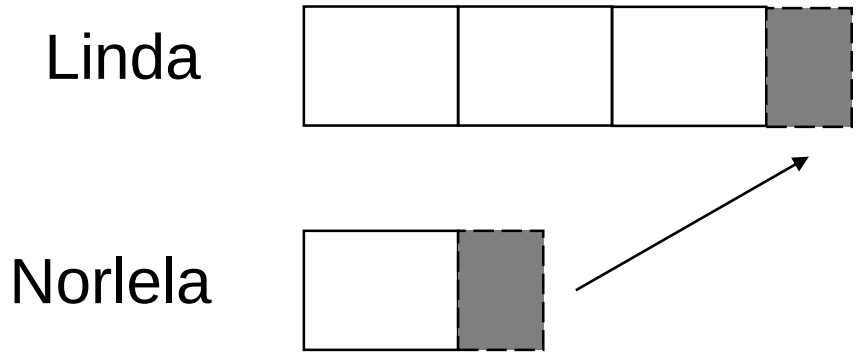
YOU DO

Linda and Norlela shared a number of posters in the ratio 3: 1.
Norlela gave Linda 63 of her posters.
Linda had 7 times as many posters as Norlela in the end.
How many posters had Linda in the end?

Before Linda : Norlela

3 : 1

6 : 2 = 8u



Total Unchanged

After Linda : 63 Norlela

7 : 1 = 8u

2u – 1u = 1u

1u = 63

7u = 63 × 7 = 441

Linda had **441** posters in the end.

Before	Linda	:	Norlela	Total
	3	:	1	4
	+63		-63	
	7	:	1	8

After	Linda	:	Norlela		
	3	:	1	4	
	(x2)		(x2)	(x2)	
	6		2		
	+63		-63		
	7	:	1	8	

1u = 63
7u = 63 × 7 = 441
Linda had **441** posters in the end.

P5/P6 Heuristics Approach
Constant Difference

WE DO

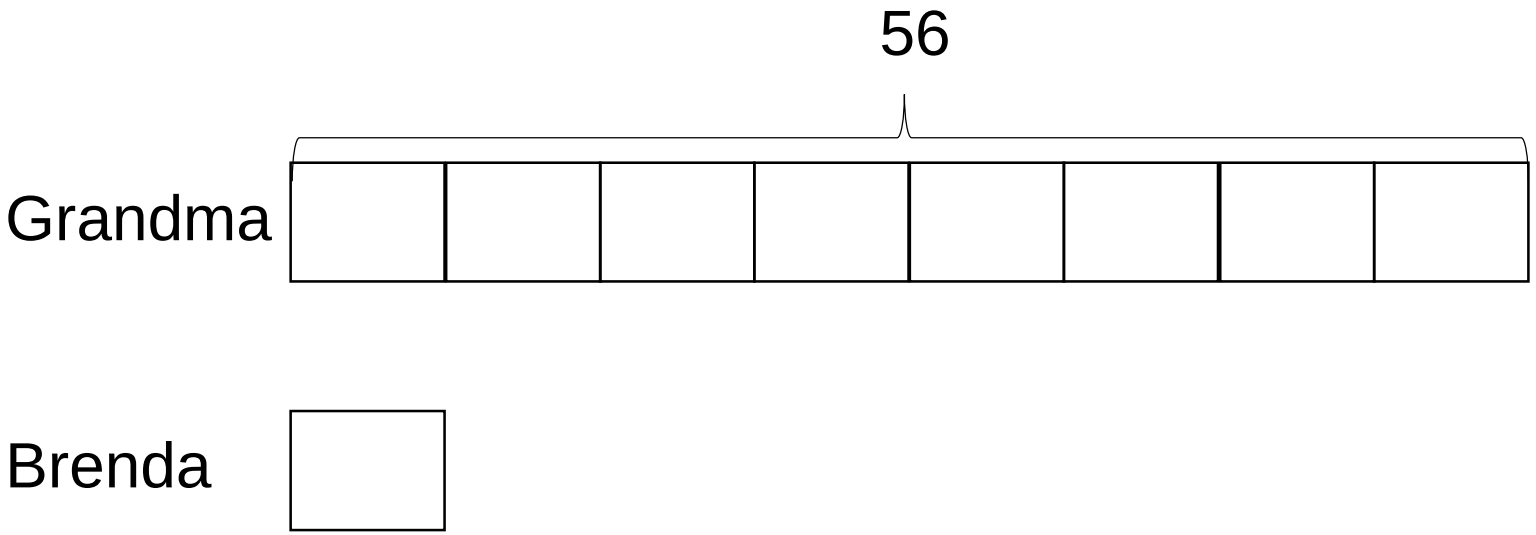
Brenda is 56 years younger than her grandmother.

In four years' time, Brenda's grandmother will be 8 times as old as Brenda.

How old is Brenda now?

Tip: The difference in their age will always be 56.

In 4 years' time



7 units = 56

1 unit = 56 ÷ 7

= 8

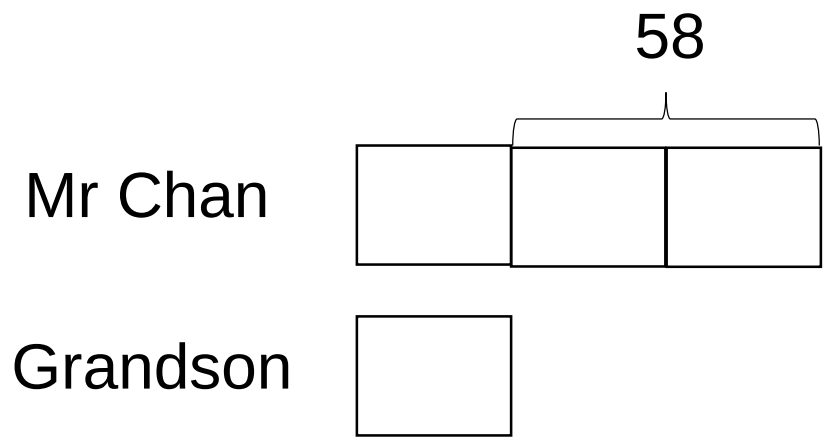
8 – 4 = 4

Brenda is **4 years old** now.

WE DO

Mr Chan is 67 years old while his grandson is 9 years old.
In how many years' time will Mr Chan be 3 times as old as his grandson?

67 – 9 = 58 (the difference in their age)



2 units = 58

1 unit = 58 ÷ 2

= 29

29 – 9 = 20

In **<u>20 years' time</u>**, Mr Chan will be as 3 times as old as his grandson.

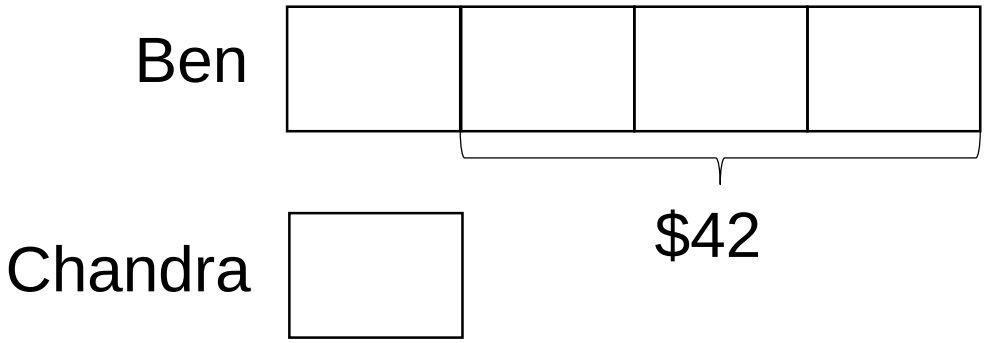
YOU DO

At first, Ben had $90 and Chandra had $48.
Each bought a shirt at the same price.
The amounts of money Ben and Chandra had left were in the ratio 4 : 1.
How much did the shirt cost?

PSLE 2015

Difference in amount of money Ben and Chandra had

→ $90 - $48 = $42



3 units = $42

1 unit = $42 ÷ 3

= $14

Cost of shirt → $48 - $14

= **$34**

P5/P6 Heuristics Approach

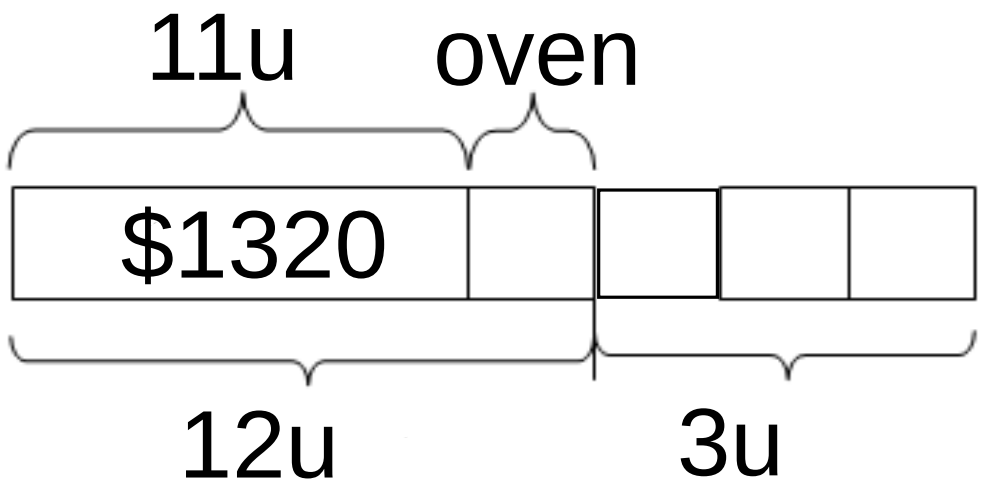
Working Backwards

WE DO

Mr Tan spent $1320 on a LCD television set and $\frac{1}{4}$ of his remaining money on an oven.

He then had $\frac{1}{5}$ of his money left.

How much money had Mr Tan at first ?

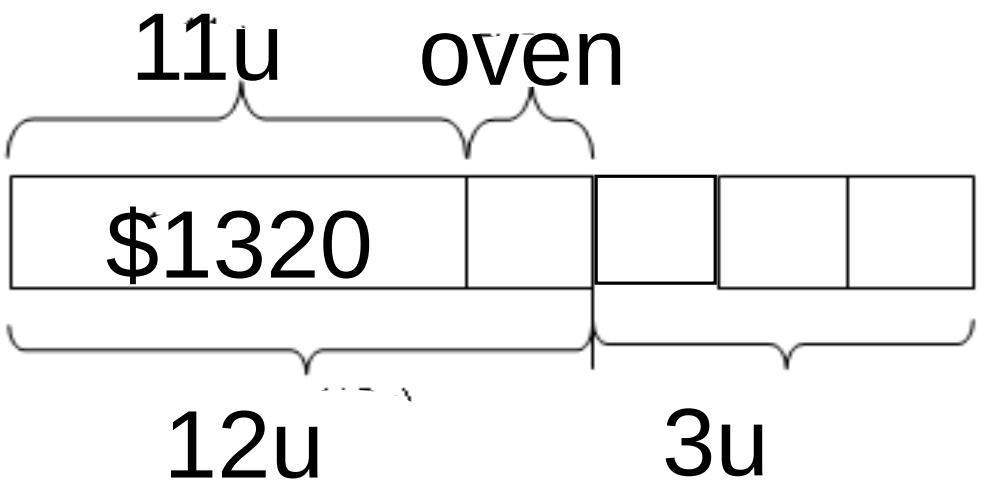


$\frac{1}{5}$ → 3u

$\frac{4}{5}$ → 3u x 4 = 12u

11u → $1320

1u → $1320 ÷ 11 = $120



12 x 15 = 1800

Mr Tan had **$1800** at first

WE DO

Mrs Tan gave Peter and Jane some money each on Sunday.

On Monday, Peter gave $\frac{1}{3}$ of the money he had to Jane.

On Tuesday, Jane gave $\frac{1}{3}$ of the money she had to Peter.

In the end, Peter had $96 and Jane had $120.

How much did each of them have at first?

In the end, Peter had $96 and Jane had $120

Jane had $120 after giving Peter $\frac{1}{3}$ of the money,

2 u = $120

1u = $60

So we take $60 from Peter and add $60 back to Jane.

Jane would have $120 + $60 = $180

Peter ---- $96 - $60 = $36.

Peter had $36 after he gave Jane $\frac{1}{3}$ of the money

Peter --- 2u = $36

1u = $18

So Peter had $36 + $18 = $54

Jane had $180 - $18 = $162

YOU DO

Ling and Juni made greeting cards over two days.

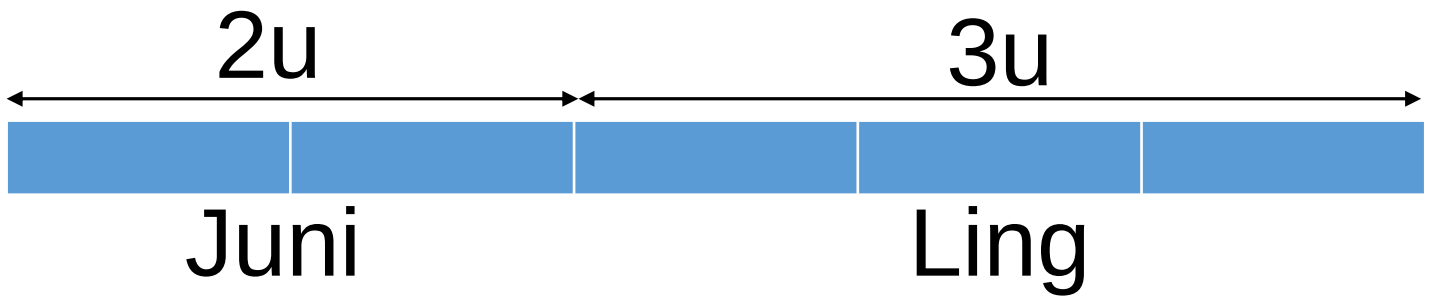
On Saturday, Ling made 19 cards more than Juni.

On Sunday, Ling made another 20 cards and Juni made another 15.

At the end of the two days, Ling made $\frac{3}{5}$ of the total number of cards.

What was the number of cards Juni made?

PSLE 2015



1 unit = 19 + 20 – 15 = 24

2 units = 2 x 24 = **48**

P5/P6 Heuristics Approach

Guess and Check

Supposition Method

WE DO

There were 115 cars, vans and motorcycles altogether in a car park. There were 4 times as many cars as vans. If these vehicles had 430 wheels altogether, how many vans were there in the car park?

No. of cars	No. of car wheels	No. of vans	No. of van wheels	No. of motor-cycles	No. of motorcycle wheels	Total no. of wheels	check
44	44 × 4 = 176	11	11 × 4 = 44	60	60 × 2 = 120	176 + 44 + 120 = 340	x
48	48 × 4 = 192	12	12 × 4 = 48	55	55 × 2 = 110	192 + 48 + 110 = 350	x
80	80 × 4 = 320	20	20 × 4 = 80	15	15 × 2 = 30	320 + 80 + 30 = 430	✓

There were **20 vans** in the car park.

Suppose all are motor-cycles:

115 x 2 = 230 (wheels)

430 – 230 = 200 (wheels -- cars and vans)

200 ÷ 2 = 100 (No of cars and vans)

There were 4 times as many cars as vans.

100 ÷ 5 = 20

There were **20 vans** in the car park.

WE DO

Bernard bought some 50-cent and 80-cent stamps for $30.60.

There were twice as many 50-cent as 80-cent stamps.

How many 80-cent stamps are there?

No. of 50-cent stamps	Amount	No. of 80-cent stamps	Amount		Total value		Check
20	20 x 0.50 = $10	10	10 x 0.80 = $8		$10 + $8 = $18		X
30	30 x 0.50 = $15	15	15 x 0.80 = $12		$15 + $12 = $27		X
34	34 x 0.50 = $17	17	17 x 0.80 = $13.60		$17 + $13.60 = $30.60		√

There are **17** (80-cent stamps).

YOU DO

Pamela spent $86 on some similar pens and some pencils.
She bought 11 more pens than pencils.
Each pen cost $2.10 and each pencil cost $1.60.

(a) How many pencils did she buy?

(b) How many pens did she buy?

Let's Check……

No. of pens	Cost	No. of pencils	Cost	Total Cost	Check
21	21 × $2.10 = $44.10	10	10 × $1.60 = $16	$44.10 + $16 = $60.10	x
26	26 × $2.10 = $54.60	15	15 × $1.60 = $24	$54.60 + $24 = $78.60	x
28	28 × $2.10 = $58.80	17	17 × $1.60 = $27.20	$58.80 + $27.20 = $86.00	✓

WE DO

A farmer has 60 chickens and cows. There are 144 legs together. How many cows are there?

Step 1 : Suppose a fixed quantity of 1 item

Suppose all are chickens

Step 2 : Multiply to find the total number of legs

60 x 2 = 120 (legs)

Step 3 : Find the extra

144 - 120 = 24 (extra legs)

Step 4 : Find the difference

4 - 2 = 2 (2 more legs to make a cow)

Step 5 : Solve (Opposite)

24 ÷ 2 = 12

12 cows

WE DO

In a Mathematics test, pupils had to solve 30 questions.
For each correct answer, 5 marks were awarded and for each incorrect answer,
2 marks were deducted.
Jason scored 73marks.
How many incorrect answers did Jason have ?

WE DO

Assume all questions are correct

$30 \times 5 = 150$

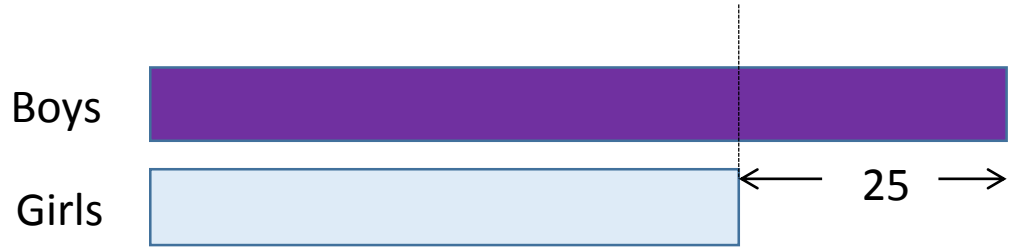
$150 - 73 = 77$ (Difference)

$5 - (-2) = 5 + 2 = 7$

$77 \div 7 = 11$ (Wrong)

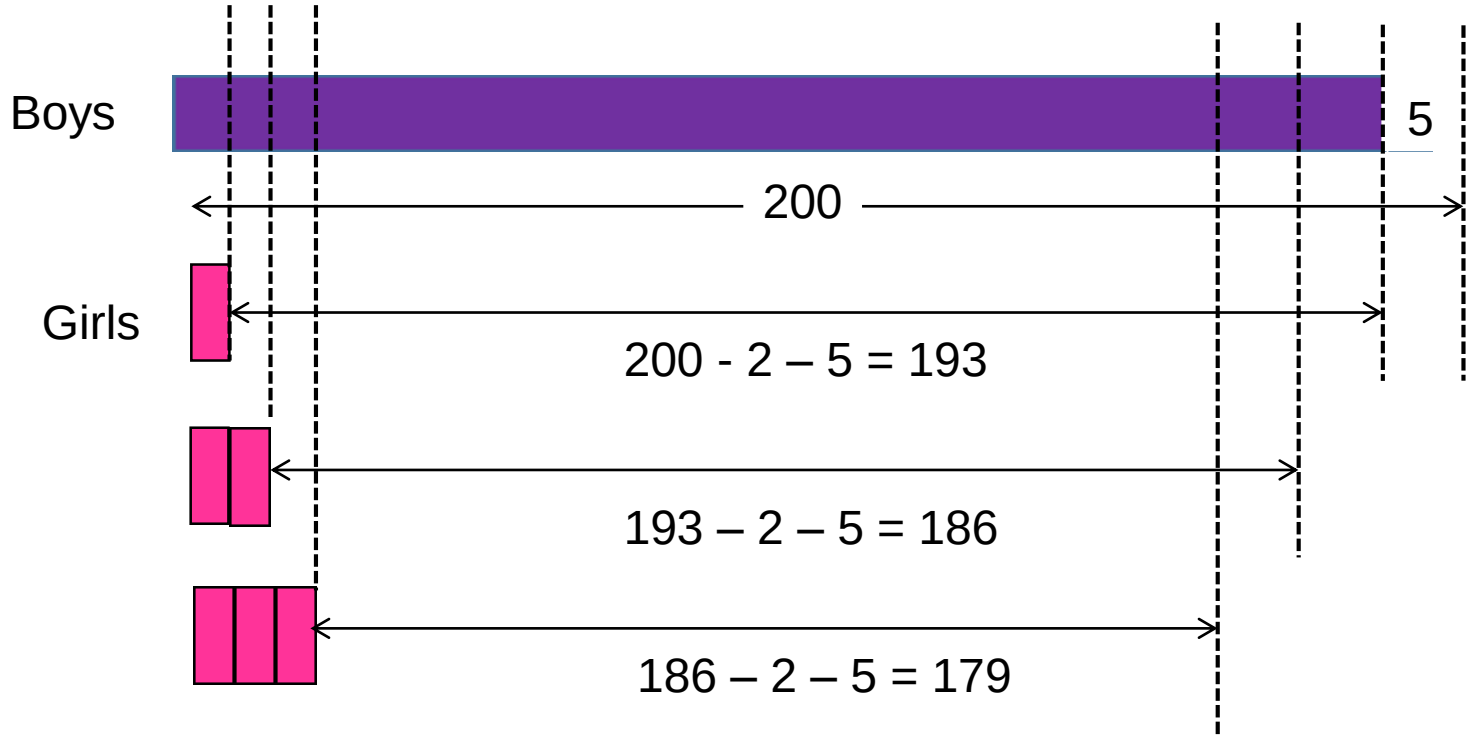
Another Example (P6)……

On Children's Day, total of 40 pupils were given some jellybeans. Each boy received 5 jellybeans and each girl received 2 jellybeans. If the boys received 25 more jellybeans than the girls, how many boys are there?



Assume all are boys, so the total number of jellybeans is [40 x 5] **= 200.**

200 – 0 = 200 (difference between boys and girls. However, the difference should be only 25 as stated in the question. So there is an extra of 175 (200 – 25) which needs to be closed up.



Each time a boy is replaced with a girl, the gap is reduced by 7

Difference is 2 + 5 = 7

So, how many times must this be done to remove 175 gaps?

Simply : 175 ÷ 7 = 25 (girls)
40 – 25 = 15 (boys)

YOU DO

Mr Lim has some kittens, white mice and birds in his pet shop.

- There are thrice as many birds as white mice.
- If these pets have 92 heads and 266 legs altogether, how many kittens are there in the shop?

Step 1 : Suppose a fixed quantity of 1 item

Assume all are birds

Step 2 : Multiply to find the total number of legs

92 x 2 = 184

Step 3 : Find the extra

266 - 184 = 82 (extra legs)

Step 4 : Find the difference

4 - 2 = 2

Step 5 : Solve (Opposite)

82 ÷ 2 = 41 (kittens and mice)

92 – 41 = 51 (birds)

51 ÷ 3 = 17 (mice)

92 – 51 – 17 = ***24*** (kittens)

THANK YOU...

Kindly complete the Workshop Evaluation.
Have a nice weekend!